STSB6816 Test 1 of 2022

Mathematical Statistics and Actuarial Science; University of the Free State

2022/04/07

## Time: 180 minutes; Marks: 50

# Instructions

* Answer all questions in a single R Markdown document. Please knit to PDF or Word at the end and submit both the PDF/Word document and the .Rmd file for assessment, in that order.
* Label questions clearly, as it is done on this question paper.
* All results accurate to about 3 decimal places.
* Show all derivations, formulas, code, sources and reasoning.
* Intervals should cover 95% probability unless stated otherwise.
* No communication software, no devices, and no communication capable websites may be accessed prior to submission. You may not (nor even appear to) attempt to communicate or pass information to another student.

# Introduction

The data is provided on <https://ufs.blackboard.com>. **It consists of the queueing times of customers at two cashiers in a small store. Your task today is to establish whether the two cashiers are similar in serving speed, or whether there is a meaningful difference between them on average.**

These times (in minutes) are assumed to follow a gamma distribution (on the basis that they are the sum of exponential waiting times of independent customers arriving uniformly over time). You are encouraged to fit gamma distributions to these times as part of this process, although other approaches will get partial credit.

# Question 1

**1.1)** Before touching the data, derive the Jeffreys prior for the gamma distribution parameters: shape $α$ and rate $λ$ (or scale $β$ if you prefer). **[ 5 ]**

**1.2)** Read in the data set and explore it visually. You could start with a histogram or density plots and by drawing a box plot of Times against the Cashier name. Give a very short summary of what you see. **[ 5 ]**

**1.3)** Fit distributions (ideally gamma distributions) to the times as a whole, as well as to the times of each individual cashier. Give parameter estimates, with uncertainty, for your fits. For full credit you must use the Jeffreys prior: $λ^{−1}\sqrt{αψ′\left(α\right)−1}$ (trace plots showing good convergence are highly recommended for simulation fits). **[ 16 ]**

Hint: to add non-standard terms to the log posterior in Stan you can use *target +=* and then Stan math functions. Stan math functions are similar to R math functions (identical in this case) including the *trigamma()* function $\left(ψ′\left(\right)\right)$.

**1.4)** For each fit, regardless of distribution, describe the modelled **average** wait time of customers in general. Either use a visual tool (e.g. density/box plot) or an estimate with interval or both. **[ 4 ]** [1 bonus mark for putting the three summaries in 1 table or plot]

**1.5)** What is the probability that one of the cashiers is more than a minute slower than the other on average? What does your answer imply practically? **[ 4 ]**

**1.6)** Consider a random future customer joining Alice’s line. What is the probability that they will wait longer than 10 minutes according to your model? **[ 3 ]**

**1.7)** Consider 10 random future customers, 6 in Alice’s line and 4 in Bob’s. What is the probability that the one that has to wait the longest will wait more than 10 minutes? **[ 5 ]**

Hint: You must construct sets of 10 customers by taking 6 Alice predictions and 4 Bob predictions and concatenating them, then check whether the longest wait time of the 10 is longer than 10 minutes. You must do this at least 1000 times and average the results to get a probability estimate.

**1.8)** **Briefly** explain more approaches that might allow you compare the models and possibly answer the core question of whether there is a meaningful difference in speed between the cashiers. Do not give any code or implementation, just words and references. **[ 8 ]**