Bayes Class 2

Sean van der Merwe

22/02/2022

# Bayes’ Theorem

$$p\left(θ|x\right)∝like\left(x|θ\right)p\left(θ\right)$$

$$logp\left(θ|x\right)=ℓ\left(x|θ\right)+logp\left(θ\right)+c\_{1}$$

$$ℓ=5nlogλ−λ\sum\_{i=1}^{n}x\_{i}+c\_{2}$$

$$logp\left(λ|x\right)=5nlogλ−λ\sum\_{i=1}^{n}x\_{i}−0.5logλ−0.5λ+c\_{3}$$

$$logp\left(λ|x\right)=\left(5n−0.5\right)logλ−λ\left(\sum\_{i=1}^{n}x\_{i}+0.5\right)+c\_{3}$$

$$λ|x∼Gamma\left(5n+0.5,∑x\_{i}+0.5\right)$$

x <- c(7, 15, 5, 11, 14, 10, 20, 10, 4)
n <- length(x)

nsims <- 10000
lambda\_sims <- rgamma(nsims, 5\*n+0.5, sum(x) + 0.5)

lambda\_sims |> density() |> plot()



rgamma(nsims, 5, lambda\_sims) -> pred\_x

hist(pred\_x)



What is the probability of waiting more than 15 minutes?

mean(pred\_x>15)

## [1] 0.1893

What is the probability that the longest wait of a random sample of 10 people is longer than 15 minutes?

lambda\_sims |> sapply(\(l) {
 rgamma(10, 5, l) |> max()
}) -> maximums
mean(maximums > 15)

## [1] 0.7935

mean(pred\_x)

## [1] 10.8037

quantile(pred\_x, c(0.025, 0.975))

## 2.5% 97.5%
## 3.354633 22.783662