Bayes Assignment 4 of 2025 by Sean van der Merwe, UFS

In this assignment you will conduct a simulation study to compare two priors in terms of their performance on a particular task. The scenario is that a set of proportions $\left(x\_{1},…,x\_{n}\right)$ are measured and a decision must be made whether they are close enough to uniform or adhering to a beta distribution with parameter values away from 1. The procedure is that a beta distribution is fitted to the proportions, and if the posterior median estimates of the parameters fall in the region $\left(h\_{1}<a<h\_{2}\right) ⋂ \left(h\_{3}<b<h4\right)$ where $h\_{1},…,h\_{4}$ are close to 1 then the conclusion is made that they are ‘close enough to uniform’. If you incorrectly conclude they are close enough then the loss is $h\_{5}$; while if you incorrectly conclude they are too far away then the loss is $h\_{6}$. Correct conclusions make a profit of $h\_{7}$.

From the history, it is known that samples come from either a $beta\left(1,1\right)$ (uniform) distribution with probability $h\_{8}$, or a $beta\left(h\_{9},h\_{10}\right)$ distribution with probability $1-h\_{8}$. The two priors being compared are **(A)** the objective prior $π\left(a,b\right)∝a^{-1}b^{-1}$ and **(B)** the subjective prior $a∼lognormal\left(0,h\_{11}\right)$ and $b∼lognormal\left(0,h\_{12}\right)$.

Given the hyperparameter values $\left(h\_{1},…,h\_{12},n\right)$ next to your student number below, which prior provides the minimum risk (expected loss)? Answer this question on the basis of $M$ simulated samples.

| Student\_num | h1 | h2 | h3 | h4 | h5 | h6 | h7 |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 2014095653 | 0.96 | 1.08 | 0.96 | 1.09 | 315 | 442 | 112 |
| 2017159092 | 0.93 | 1.07 | 0.95 | 1.06 | 263 | 365 | 144 |
| 2017418365 | 0.95 | 1.06 | 0.97 | 1.06 | 388 | 326 | 151 |
| 2018006516 | 0.97 | 1.07 | 0.92 | 1.04 | 267 | 382 | 143 |
| 2018395968 | 0.94 | 1.03 | 0.96 | 1.08 | 236 | 424 | 107 |
| 2019369780 | 0.95 | 1.06 | 0.92 | 1.04 | 270 | 483 | 130 |
| 2020231664 | 0.93 | 1.06 | 0.97 | 1.03 | 295 | 335 | 104 |
| 2021603747 | 0.91 | 1.03 | 0.94 | 1.04 | 328 | 431 | 96 |
| 2024180487 | 0.97 | 1.11 | 0.94 | 1.07 | 324 | 400 | 96 |
| 2028830517 | 0.94 | 1.10 | 0.93 | 1.05 | 366 | 397 | 133 |
| 2021234567 | 0.94 | 1.06 | 0.96 | 1.08 | 324 | 429 | 152 |
| 2022345678 | 0.94 | 1.07 | 0.93 | 1.09 | 207 | 314 | 101 |
| 2023456789 | 0.95 | 1.04 | 0.93 | 1.04 | 398 | 480 | 98 |

| Student\_num | h8 | h9 | h10 | h11 | h12 | n | M |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 2014095653 | 0.32 | 1.09 | 1.16 | 2.0 | 1.7 | 50 | 1300 |
| 2017159092 | 0.40 | 0.91 | 0.86 | 1.5 | 2.0 | 43 | 1500 |
| 2017418365 | 0.62 | 1.09 | 1.13 | 1.8 | 1.9 | 36 | 1500 |
| 2018006516 | 0.42 | 0.92 | 0.88 | 2.0 | 1.9 | 41 | 1600 |
| 2018395968 | 0.34 | 1.14 | 1.17 | 1.9 | 0.9 | 41 | 1500 |
| 2019369780 | 0.46 | 0.86 | 0.81 | 1.9 | 1.7 | 37 | 1200 |
| 2020231664 | 0.36 | 1.07 | 1.18 | 1.9 | 1.2 | 37 | 1300 |
| 2021603747 | 0.67 | 0.88 | 0.85 | 0.8 | 1.1 | 46 | 1500 |
| 2024180487 | 0.70 | 1.10 | 1.18 | 1.0 | 1.8 | 49 | 1300 |
| 2028830517 | 0.40 | 0.88 | 0.81 | 1.3 | 1.3 | 45 | 1600 |
| 2021234567 | 0.68 | 1.09 | 1.15 | 2.1 | 1.5 | 45 | 1100 |
| 2022345678 | 0.69 | 0.89 | 0.85 | 1.6 | 1.5 | 46 | 1500 |
| 2023456789 | 0.48 | 1.13 | 1.19 | 0.9 | 2.0 | 44 | 1600 |

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