Bayes Main Assignment 3 of 2022

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Instructions

Two sets of patients (individual subjects) are asked to rate their pain levels following a routine operation. They record the levels daily for 16 days. The first set receives a painkiller currently on the market, the second set receives a pain treatment that should be equivalent but cheaper and needs to be tested. Neither patients nor doctors know which patient receives which. Your goal is to determine whether the new treatment is as least 90% as effective as the one currently on the market.

The measurements are perceptions rated on a scale of 0 to 10.

Prior studies have suggested that the pain is sharp at first, dropping quickly to a more shallow pain that lingers and fades slowly. Suggested models include an exponential slope $\left(\mu e^{-\lambda x}\right)$ with priors $\mu \sim Beta(5.5,2.5)$ and $\lambda \sim Beta(1.5,5.5)$.

Part 1

Read in and visualise your data. Is there an observable difference between the two treatments in the patterns observed?

HINT Convert the data to long format before removing the missing values: Create a new data set where the columns for the 16 days are converted to 2 columns: *Day* and *Score*. The other needed columns will have to be stretched to accommodate the longer form of the data. Consider using the *pivot_longer* command.

Part 2

A Binomial GLM formulation is recommended as a starting point. Ignore the time and subject effects for now, just assume i.i.d. observations first (conditional on treatment of course). What is the modelled treatment effect? Can you give a value for $P\left[\frac{\text{Treatment 2 effect}}{\text{Treatment 1 effect}} > 0.9\right]$ or something similar?

Part 3

Bring in the subject effect. Consider each subject to have a random effect, but with constant variances throughout. How does this change your previous conclusions and results?

Part 4

Bring in the time effect neatly, ideally assuming an exponential decay (fast drop then slow drop in pain levels) although a linear decay will work nearly as well. Which treatment is better after 1 day and which treatment is better after 10 days?

Incorporating the subject effect is not required for full marks, but will earn bonus marks, especially if you can answer the above question with uncertainty for a random future person.

Part 5

Compare your previous models using at least one criterion and say which model seems the most parsimonious. The options (in order of preference) include LOOIC, WAIC, mDIC, cDIC (standard DIC). Properly calculated Bayes Factors would also get full credit.

[Each of the 5 parts above is worth 20 marks. Total is out of 100.]